Two-Dimensional Viewing
Viewing Pipeline

Modeling Coordinates ➔ World Coordinates ➔ Viewing and Projection Coordinates ➔ Normalized Coordinates ➔ Video Monitor

Plotter

Other Output

Device Coordinates
Two-Dimensional Viewing

- Two dimensional viewing transformation
  - From world coordinate scene description to device (screen) coordinates
Normalization and Viewport Transformation

- World coordinate clipping window
- Normalization square: usually $[-1,1] \times [-1,1]$
- Device coordinate viewport

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A point $(xw, yw)$ in a clipping window is mapped to a normalized coordinate position $(x_{\text{norm}}, y_{\text{norm}})$, then to a screen-coordinate position $(xv, yv)$ in a viewport. Objects are clipped against the normalization square before the transformation to viewport coordinates.
Clipping

- Remove portion of line outside viewport or screen boundaries

- Two approaches:
  - Clip during scan conversion: per-pixel bounds check, or span endpoint tests.
  - Clip analytically, then scan-convert the modified primitive.
Two-Dimensional Clipping

- Point clipping – trivial
- Line clipping
  - Cohen-Sutherland
  - Cyrus-beck
  - Liang-Barsky
- Fill-area clipping
  - Sutherland-Hodgeman
  - Weiler-Atherton
- Curve clipping
- Text clipping
Line Clipping

- Basic calculations:
  - Is an endpoint inside or outside the clip rectangle?
  - Find the point of intersection, if any, between a line segment and an edge of the clip rectangle.

- Both endpoints inside ✅ trivial accept
- One inside ✅ find intersection and clip
- Both outside ✅ either clip or reject
## Cohen-Sutherland Line-Clipping Algorithm

<table>
<thead>
<tr>
<th>Region code for each endpoint</th>
<th>Bit 4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>above</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>below</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>right</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>left</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

### View port

- 1001
- 0001
- 0101
- 1000
- 0000
- 0100
- 1010
- 0010
- 0110
Cohen-Sutherland Line-Clipping Algorithm

- Trivially accepted
  - if (both region codes = 0000)
- Trivially rejected
  - if (AND of region codes ≠ 0000)
- Otherwise, divide line into two segments
  - test intersection edges in a fixed order.
    - (e.g., top-to-bottom, right-to-left)
Cohen-Sutherland Line-Clipping Algorithm

* Fixed order testing and clipping cause needless clipping (external intersection)
Cohen-Sutherland Line-Clipping Algorithm

- Midpoint Subdivision for locating intersections
  1. trivial accept/reject test
  2. midpoint subdivision:
     \[ x_m = (x_1 + x_2)/2, \quad y_m = (y_1 + y_2)/2 \]
     (one addition and one shift)
  3. repeat step 1 with two halves of line
     \[ \Rightarrow \text{good for hardware implementation} \]
Cohen-Sutherland
Line-Clipping Algorithm

- When this is good
  - If it can trivially reject most cases
  - Works well if a window is large w.r.t. to data
  - Works well if a window is small w.r.t. to data
  - i.e., it works well in extreme cases
  - Good for hardware implementation
Use a parametric line equation

\[ P(t) = P_0 + t(P_1 - P_0), \quad 0 \leq t \leq 1 \]

Reduce the number of calculating intersections by simple comparisons of parameter $t$. 

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The page contains a section on **Parametric Line Clipping (Cyrus-beck Technique)**. It explains the use of a parametric line equation and how to reduce the number of calculating intersections by simple comparisons of the parameter $t$. The equation given is:

\[ P(t) = P_0 + t(P_1 - P_0), \quad 0 \leq t \leq 1 \]
Parametric Line Clipping
(Cyrus-beck Technique)

Algorithm

- For each edge $E_i$ of the clip region
- $N_i$: outward normal of $E_i$
Choose an arbitrary point $P_{E_i}$ on edge $E_i$ and consider three vectors $P(t) - P_{E_i}$

$\Rightarrow$

$N_i \cdot (P(t) - P_{E_i}) < 0 \iff$ a point in the side halfplane
$N_i \cdot (P(t) - P_{E_i}) = 0 \iff$ a point on the line containing the edge
$N_i \cdot (P(t) - P_{E_i}) > 0 \iff$ a point in the outside halfplane
Parametric Line Clipping (Cyrus-beck Technique)

- Solve for the value of $t$ at the intersection of $P_0P_1$ with the edge:
  \[ N_i \cdot [P(t) - P_{Ei}] = 0. \]
  \[ P(t) = P_0 + t(P_1 - P_0) \]
  and let $D = (P_1 - P_0)$.

Then

\[ t = \frac{N_i \cdot [P_0 - P_{Ei}]}{-N_i \cdot D} \]

$N_i \neq 0$,

$D \neq 0$ (that is $P_0 \neq P_1$),

$N_i \cdot D \neq 0$ (if not, no intersection)
Parametric Line Clipping (Cyrus-beck Technique)

- Given the four values of $t$ for a line segment, determine which pair of $t$'s are internal intersections.

  If $t \notin [0,1]$ then discard
  else choose a (PE, PL) pair that defines the clipped line.

- PE(potentially entering) intersection:

  if moving from $P_0$ to $P_1$ causes us to cross an edge to enter the edge's inside half plane;
Parametric Line Clipping (Cyrus-beck Technique)

- PL (potentially leaving) intersection:
  - if moving from $P_0$ to $P_1$ causes us to leave the edge's inside half plane.

  \[
  \begin{align*}
  \text{i.e., } N_i \overline{P_0P_1} &< 0 \implies \text{PE} \\
  N_i \overline{P_0P_1} &> 0 \implies \text{PL}
  \end{align*}
  \]

- Intersections can be categorized!
- Inside the clip rectangle $(T_E, T_L)$
  - $T_E$: select PE with largest $t$ value $\geq 0$
  - $T_L$: select PL with the smallest $t$ value $\leq 1$. 
Parametric Line Clipping (Cyrus-beck Technique)

- This is an efficient algorithm when many line segments need to be clipped
- Can be extended easily to convex polygon windows
Liang-Barsky line clipping

- The ideas for clipping line of Liang-Barsky and Cyrus-Beck are the same. The only difference is Liang-Barsky algorithm has been optimized for an upright rectangular clip window.

- Finds the appropriate end points with more efficient computations.
Let PQ be the line which we want to study.

Parametric equation of the line segment

\[ x = x_1 + (x_2 - x_1)t = x_1 + dx \times t \]
\[ y = y_1 + (y_2 - y_1)t = y_1 + dy \times t \]

\[ t = 0 \Rightarrow P(x_1, y_1) \]
\[ t = 1 \Rightarrow Q(x_2, y_2) \]
Liang-Barsky Line Clipping

1. Set $t_{\text{min}} = 0$ and $t_{\text{max}} = 1$

2. Calculate the values of $t_T$, $t_B$, $t_L$, $t_R$,

- **Top edge:** $y = T$
  
  
  \[ y_1 + t^*(y_2 - y_1) = T \]

  \[ t_T = \frac{T - y_1}{y_2 - y_1} \]

- **Bottom edge:** $y = B$
  
  \[ y_1 + t^*(y_2 - y_1) = B \]

  \[ t_B = \frac{B - y_1}{y_2 - y_1} \]

- **Left edge:** $x = L$
  
  \[ x_1 + t^*(x_2 - x_1) = L \]

  \[ t_L = \frac{L - x_1}{x_2 - x_1} \]

- **Right edge:** $x = R$
  
  \[ x_1 + t^*(x_2 - x_1) = R \]

  \[ t_R = \frac{R - x_1}{x_2 - x_1} \]
Liang-Barsky Line Clipping

- If \( t < t_{\text{min}} \) or \( t > t_{\text{max}} \), ignore it and go to the next edge.

- Otherwise classify the \( t \) value as entering or exiting value (using the inner product to classify):
  - Let \( PQ \) be the line and \( N \) is normal vector
  - If \( N \cdot (Q - P) \leq 0 \), the parameter \( t \) is entering
  - If \( N \cdot (Q - P) > 0 \), the parameter \( t \) is exiting

- If \( t \) is entering value, set \( t_{\text{min}} = t \), if \( t \) is exiting value set \( t_{\text{max}} = t \)
3. If $t_{\text{min}} < t_{\text{max}}$ then draw a line from $(x_1 + dx \times t_{\text{min}}, y_1 + dy \times t_{\text{min}})$ to $(x_1 + dx \times t_{\text{max}}, y_1 + dy \times t_{\text{max}})$
Clipping

- Clipping rotated windows, circles
  - trivial acceptance/rejection test with respect to bounding rectangle of the window

- Line clipping using nonrectangular clip windows
  - extend Cyrus-Beck algorithm
Polygon clipping

- **Sutherland-Hodgeman Algorithm**
  - clip against 4 infinite clip edge in succession

![Diagram of polygon clipping steps](image)
Sutherland-Hodgeman Algorithm

- Accept a series of vertices (polygon) and outputs another series of vertices
- Four possible outputs

1. \( \mathbf{v}_1 \rightarrow \mathbf{v}_2 \rightarrow \mathbf{v}_1 \rightarrow \mathbf{v}_2 \rightarrow \mathbf{v}_1 \rightarrow \mathbf{v}_2 \)
   - Output: \( \mathbf{v}_1', \mathbf{v}_2 \)
2. \( \mathbf{v}_2 \rightarrow \mathbf{v}_1 \rightarrow \mathbf{v}_2 \rightarrow \mathbf{v}_1 \rightarrow \mathbf{v}_2 \rightarrow \mathbf{v}_1 \)
   - Output: \( \mathbf{v}_2 \)
3. \( \mathbf{v}_1 \rightarrow \mathbf{v}_2 \rightarrow \mathbf{v}_1 \rightarrow \mathbf{v}_2 \rightarrow \mathbf{v}_1 \rightarrow \mathbf{v}_2 \)
   - Output: \( \mathbf{v}_1' \)
4. \( \mathbf{v}_1 \rightarrow \mathbf{v}_2 \rightarrow \mathbf{v}_1 \rightarrow \mathbf{v}_2 \rightarrow \mathbf{v}_1 \rightarrow \mathbf{v}_2 \)
   - Output: none
The Sutherland-Hodgeman Algorithm

- The algorithm correctly clips convex polygons, but may display extraneous lines for concave polygons.

- How clip?
How to correctly clip

[Way I] Split the concave polygon into two or more convex polygons and process each convex polygon separately.

[Way II] Modify the algorithm to check the final vertex list for multiple vertex points along any clip window boundary and correctly join pairs of vertices.

[Way III] Use a more general polygon clipper
Clipping concave polygons

- Split the concave polygon into two or more convex polygons and process each convex polygon separately.
  - vector method for splitting concave polygons
    ⇒ calculate edge-vector cross products in a counterclockwise order. If any $z$ component turns out to be negative, the polygon is concave.
Weiler-Atherton Polygon Clipping

- For an outside-to-inside pair of vertices, follow the polygon boundary.
- For an inside-to-outside pair of vertices, follow the window boundary in a clockwise direction.
Weiler-Atherton Polygon Clipping

- Polygon clipping using nonrectangular polygon clip windows

![Diagram of polygon clipping](image)

Figure 6-30

Clipping a polygon fill area against a concave-polygon clipping window using the Weiler-Atherton algorithm.
Texture Clipping

1. all-or-none text clipping: Using boundary box for the entire text
2. all-or-none character clipping: Using boundary box for each individual
3. clip individual characters
   - vector: clip line segments
   - bitmap: clip individual pixels
What we have got!